Comment on "Dynamics of a Charged Particle"

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The equation derived by F.Rohrlich (Phys.Rev.E **77**, 046609 (2008)) reproduces Eq.(76.3) from the Landau and Lifshitz book (The Classical Theory of Fields). The new validity condition for this equation is inapplicable as it is, but once fixed it coincides with Eqs.(75.11-12) in the cited book.

Dr. Rohrlich seems to be unaware of some of important previous work relevant to the subject of his paper [1].

1. I discuss the equation of the charged particle motion derived in [1] as applied to a case when the external force is created by an electromagnetic field [2]. First, this equation reproduces Eq.(76.3) from the Landau and Lifshitz book [3], with the derivation coinciding with that found in [4]. Consider the *quotations* from [4]:

"...The modified Lorentz-Dirac equation : $m\mathbf{a} = \mathbf{f}_{\rm ext} + t_0\dot{\mathbf{f}}_{\rm ext}$ *(9.8)"*. Compare this with Eq.(5a) from [1]. *"...We obtain:* $ma^\alpha = f^\alpha_{\rm ext} + t_0(\delta^\alpha{}_\beta + u^\alpha u_\beta) f^\beta_{{\rm ext}\,,\gamma} u^\gamma$ *(9.9) - the relativistic version of the modified Lorentz-Dirac equation"*. Compare this with the ultimate Eq.(14) from [1] - the only difference is the use of $f^\beta_{{\rm ext}\,,\gamma} u^\gamma = \dot{f}^\beta_{\rm ext}$.

The cited derivation found in [4] is not presented as original work, but, as being derived following [3]. Among the problems to be solved by students this survey suggests the following: "**Problem 16**. *Show that if the external force is provided by an external electromagnetic field* $F^{\alpha\beta}_{\rm ext}$, *then the modified Lorentz-Dirac equation takes the form:* $ma^\alpha = qF^\alpha_{{\rm ext}\ \beta} u^\beta$ $+qt_0\left[F^\alpha_{{\rm ext}\ \mu,\nu} u^\mu u^\nu + \frac{q}{m}\left(\delta^\alpha{}_\beta + u^\alpha u_\beta\right) F^\beta_{{\rm ext}\ \mu} F^\mu_{{\rm ext}\ \nu} u^\nu\right]$." The latter equation is present in [3] as Eq.(76.3).

Paper [1] does not demonstrate (as solving "Problem 16" would) that the final Eq.(14) (the same as Eq.(9.9) from [4]) is equivalent to the well-known Eq.(76.3) in [3], allowing the reader to assume that the derived Eq.(14) is new. This equivalence is evident when the convective derivative of the External Lorentz Force (ELF), $f^\alpha_{\rm ext} = qF^\alpha_{{\rm ext}\ \mu} u^\mu$, as present in Eq.(9.9) is expanded:

$$\dot{f}^\alpha_{\rm ext} = u^\gamma f^\alpha_{{\rm ext}\,,\gamma} = q\left(u^\gamma F^\alpha_{{\rm ext}\ \mu,\gamma} u^\mu + F^\alpha_{{\rm ext}\ \mu} a^\mu\right), \qquad (1)$$

where u^α and $a^\alpha = \dot{u}^\alpha$ are 4-velocity and 4-acceleration, q is the particle charge and $F^{\alpha\beta}_{\rm ext}$ is the field tensor. Now the equation to be derived (see "Problem 16") is obtained from Eq.(9.9) by: (1) using the anti-symmetry of the field tensor; and (2) expressing within the accuracy of the used approximation (neglecting terms $\propto \tau_0^2$) a^α in Eq.(1) in terms of the Lorentz force: $ma^\alpha = qF^\alpha_{{\rm ext}\ \mu} u^\mu$. Thus, no new equation of motion for a charge is derived in [1].

2. If, again, we consider the particular case of the ELF and employ our Eq.(1) in order to expand $\dot{f}^\alpha_{\rm ext}$ in the equations in [1], many of them start to look unusable.

Particularly, the requirement that "... the external force must vary slowly enough over the size of the charge distribution" in [1] is only applicable to the external *field* (not force!). Once this consideration is applied to *force*, as is done in Ineq.(1) in [1], the time derivative of the ELF (in the instantaneous rest frame) should be expanded in the way analogous to how it is done in Eq.(1)

$$\frac{d\mathbf{f}_{\rm ext}}{dt} = \lim_{|\mathbf{v}|\to 0} q\frac{d}{dt}\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}\times\mathbf{B}]\right) =$$
$$= q\frac{d\mathbf{E}}{dt} + \frac{q}{c}\left[\frac{d\mathbf{v}}{dt}\times\mathbf{B}\right] \approx q\frac{d\mathbf{E}}{dt} + \frac{q^2}{cm}[\mathbf{E}\times\mathbf{B}], \qquad (2)$$

where, again, the acceleration is expressed within the accuracy of the used approximation in terms of the ELF. Therefore, the validity condition, $\tau_0|d\mathbf{f}_{\rm ext}/dt| \ll |\mathbf{f}_{\rm ext}|$ (which is given in [1] as Ineq.(1)) actually requires that

$$\tau_0 dE/dt \ll E, \qquad \tau_0 qB/(mc) \ll 1. \qquad (3)$$

The above considerations follow Ch.75 in [3]. Specifically, "our" Eq.(2) proves the equivalence between Eq.(5a) in [1] (in which the radiation force equals $\tau_0 d\mathbf{f}_{\rm ext}/dt$), on one hand, and the radiation force as in Eq.(75.10) in [3] (expressed in terms of the right hand side of "our" Eq.(2)), on the other hand. The validity conditions Ineqs.(3) are equivalent to Eq.(75.11-12) from [3]. In contrast with the ideology of [1] (see the quotation above), not only the electric field variance, but also the magnetic field magnitude should be restrained [3].

Summary.

Approximate equations of motion and their validity conditions presented in [1] are equivalent (i.e. approximately equal within the accuracy of the used approximation) to those found in [3]. These results of [1] may be treated as the intermediate steps of presented in [3] derivations, which steps, although omitted in [3], are available in more expanded presentations, such as [4].

[1] F. Rohrlich, Phys. Rev. E **77**, 046609 (2008).
[2] I do not discuss a case of an *arbitrary* external force.
[3] L. D. Landau and E. M. Lifshits, *The Classical Theory of Fields* (Pergamon, New York, 1994).
[4] E. Poisson, *An introduction to the Lorentz-Dirac Equation*, arXiv:gr-qc/9912045 (1999); URL: http://arxiv.org